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INERTIAL MOTION OF A BODY IN AN IDEAL FLUID FROM THE STATE AT REST

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The motion of a body in an ideal incompressible fluid flow without vortices in the absence of external forces is considered. It is demonstrated that the body can move inertially from the state at rest if its shape satisfies certain conditions.

Key words: *ideal incompressible fluid, vortex-free flow, motion of a body in a fluid, inertia.*

Introduction. The first studies of motion of various bodies in fluids under the action of internal forces were performed with modeling the motion of living organisms in water. Models of fish motion were developed, based on models of motion of bodies in a vortex-free flow of an inviscid fluid. Such a model of motion due to finite deformations of the body in a fluid was proposed in [1].

The general features of motion of deformable bodies in an ideal fluid from the state at rest were summarized in [2]. The possibility of motion of the body in an ideal fluid due to body deformations and changes in the mass distribution inside the body was verified in [3]. Examples of motion of a pulsing sphere and an ellipsoid with a variable eccentricity and examples of motion of a body due to small deformations defined by two parameters were also given in [3].

The motion of a deformable body in a fluid can be described by the Lagrangian equations. Based on these equations, axisymmetric and plane flows around bodies were studied, and conditions were found, which restrict the possibility of body motion for arbitrary given laws of variation of the body shape and changes in the mass distribution inside the body [4].

A problem of motion of a nondeformable body with variable internal characteristics in an ideal fluid was considered in [5]. It follows from the known solutions that the body displacement in a fluid can be provided by periodic changes in parameters determining the body shape. The motion is nonuniform: the translational velocity is not constant, and the body stops if internal forces cease to act and deformation is terminated (in the case of a deformable body). Until now, the theory of self-induced motion of a body in a fluid predicted that it is impossible to reach uniform motion on the basis of available solutions of the problem of self-induced motion of the body [3]. It is demonstrated below that a body, being at rest at the initial time, can uniformly move in an ideal fluid under the action of internal forces.

1. Shape of the Body that can Ensure its Inertial Motion in a Fluid from the State at Rest. Let us consider the motion of a body in an unbounded ideal incompressible fluid with the state at rest at infinity. The fluid motion does not contain vortices, and there are no external forces. The motion in the gravity field with neutral buoyancy of the body also refers to this case. The body is at rest at the initial time.

Let us find the body shape that can ensure inertial motion of the body from the state at rest. We consider arbitrary symmetric shapes. The symmetry of the body surface is necessary for solutions of dynamic equations corresponding to straight-line motion of the body to exist. It is known that there are no solutions of this problem for axisymmetric or cylindrical body surfaces; therefore, we analyze a three-dimensional fluid flow around the body.

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We consider a body whose surface S has an axis of symmetry of the Nth order. This means that S coincides with itself after its turning around the axis of symmetry by an angle $2\pi/N$, where $N = 2, 3, \ldots$. An example of a body with this type of symmetry is a ship screw for which the number N equals the number of blades. For $N = 2$, such a symmetry is inherent in a three-axial ellipsoid.

We study symmetric motions of the body at which the axis of symmetry of the body coincides with the x axis. Obviously, such motions are possible by virtue of symmetry of the body and unboundedness of the fluid. We consider a body whose surface S does not experience deformations induced by the outer solid shell. Inside the body, there is another solid of smaller size, which is symmetric with respect to the x axis and can rotate around the latter. The center of mass of the body is motionless with respect to the surface S and is located on the x axis.

The body and its location in space are defined by three generalized coordinates: the coordinate x_0 of the center of mass of the body on the x axis, the angle φ of turning of the body surface around the x axis, and the angle ψ of turning of the inner body around the x axis.

The motion of a solid in a potential flow of an ideal fluid is described by the Lagrangian equations of the second kind [6]. Correspondingly, in the absence of external forces, the equations of motion of the body–fluid system considered are determined by the Lagrangian function equal to the kinetic energy of the system T:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i, \qquad i = 1, 2, 3. \tag{1}$$

Here $q_1 = x_0$, $q_2 = \varphi$, and $q_3 = \psi$. The generalized force Q_1 equals zero, and the generalized forces Q_2 and Q_3 are determined by the moment M of internal forces with respect to the x axis, which acts on the inner body:

$$Q_2 = -M, \qquad Q_3 = M.$$

The kinetic energy of the system T depends on three velocities:

$$T = T(u, \omega, \Omega), \qquad u = \dot{x}_0, \qquad \omega = \dot{\varphi}, \qquad \Omega = \dot{\psi}$$

(u is the translational velocity of the body and ω and Ω are the angular velocities of the solid shell and the inner body, respectively). The kinetic energy of the system T equals the sum of the kinetic energies of the fluid T_f and the body T_b:

$$T = T_f + T_b; \tag{2}$$

$$T_f = m_f u^2/2 + I_f \omega^2/2 + K\omega u; \tag{3}$$

$$T_b = m_b u^2/2 + I_0 \omega^2/2 + I_* \Omega^2/2. \tag{4}$$

Here the coefficients of the kinetic energy of the fluid m_f, K, and I_f depend on the body shape and diameter and are proportional to the fluid density, the coefficient m_b is the total mass of the body including the inner body, and I_0 and I_* are the moments of inertia of the solid shell of the body and the inner solid, respectively. The coefficients of the quadratic shapes (3) and (4) are independent of coordinates and time. Owing to unboundedness of the fluid, its kinetic energy does not depend on the translational coordinate.

We consider nontrivial shapes of three-dimensional symmetric bodies with the coefficient K in Eq. (3) other than zero [e.g., for screw-shaped bodies (see Sec. 2)]. Note that $K = 0$ for a three-axial ellipsoid, which is not considered in the present paper.

The Lagrangian equations (1) can be written as

$$\frac{d}{dt}\frac{\partial T}{\partial u} = 0, \qquad \frac{d}{dt}\frac{\partial T}{\partial \omega} = -M; \tag{5}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \Omega} = M. \tag{6}$$

As the body is at rest at the initial time $t = 0$, the initial conditions have the form

$$u = 0, \qquad \omega = \Omega = 0, \qquad t = 0. \tag{7}$$

Equations (5)–(7) yield the integrals of the problem of motion of a heterogeneous body in the fluid

$$\frac{\partial T}{\partial u} = 0, \qquad \frac{\partial T}{\partial \omega} + \frac{\partial T}{\partial \Omega} = 0. \tag{8}$$

Relations (8) are the laws of conservation of momentum and the moment of momentum for motion of the system from the state at rest.

From Eqs. (2)–(4) and (8), we find the translational velocity of the body

$$u = KI_*\Omega/D, \qquad D = (m_f + m_b)(I_0 + I_f) - K^2. \tag{9}$$

As the kinetic energy of the fluid determined by Eq. (3) is non-negative ($T_f \geqslant 0$), then, we have $m_f I_f \geqslant K^2$. Hence, $D > 0$.

According to Eq. (9), the translational velocity u of the body is proportional to the angular velocity Ω of rotation of the inner solid. Therefore, the body displacement is a linear function of the angle of turning of the inner solid.

From Eqs. (2)–(4), (6), and (7), we find the internal moment of momentum:

$$I_*\Omega = \int_0^t M\,dt. \tag{10}$$

We assume that the moment of internal forces $M(t)$ differs from zero only on a limited interval of time $0 < t < \tau$ and $M(t) = 0$ for $t > \tau$. For $t = \tau$, the value of the integral in the right side of Eq. (10) can be assumed to be arbitrary (in the present paper, we assume that this value differs from zero).

From Eqs. (9) and (10), we find the translational velocity

$$u = \text{const} \neq 0, \qquad t \geqslant \tau. \tag{11}$$

Relation (11) means that the body moves uniformly along the x axis.

We demonstrated that uniform motion of a body in an ideal fluid is possible under the action of internal forces at the initial interval of time $(0, \tau)$.

Thus, we proved a theorem about the body shape that can ensure inertial motion of the body from the state at rest. The theorem has the following conditions: 1) the body surface has an axis of symmetry of the Nth order; 2) for body motion along this axis, the expression for the kinetic energy of the fluid contains a non-zero contribution of the product of the translational and angular velocities. For the body with the indicated shape to move inertially, two other conditions have to be satisfied: 3) presence of one symmetric body on the axis; 4) uniform rotation of this solid body.

2. Example of the Shape of a Three-Dimensional Body and Calculation of the Coefficients of the Kinetic Energy. As an example, we consider a body with an axis of symmetry of the second order (N=2) consisting of a sphere and two thin circular disks. Figure 1 shows the projection of this body onto a plane normal to the disks and containing the axis of symmetry. Let us denote the disk radius by a and the sphere radius by R. The sphere is rigidly connected with the disks by two solid rods orthogonal to the x axis and lying in the planes of the disks. We use α ($0 < \alpha < \pi/2$) to denote the angle between the disk and the plane orthogonal to the x axis.

To calculate the kinetic energy of the fluid, we introduce the following constraints on the geometric parameters: the rod diameter d is small ($d \ll R$ and $d \ll a$); the distance h from the center of the sphere to the center of each disk is large ($h \gg R$ and $h \gg a$). As it follows from the analysis of the hydrodynamic problem by the method of perturbations, under these constraints, the allowance for the hydrodynamic interaction of the sphere and disks yields small corrections to the kinetic energy of the order of $(R/h)^3$ and $(a/h)^3$. In the main approximation with respect to two small parameters, the hydrodynamic interaction is insignificant.

Let us write the expression for the kinetic energy of an ideal fluid of density ρ for an isolated sphere moving with a velocity u:

$$T_0 = (1/3)\pi\rho R^3 u^2. \tag{12}$$

In the case of motion of an isolated thin disk, which is a particular case of an ellipsoid of revolution, the kinetic energy of the fluid [7] can be presented as

$$T_1 = (4/3)\rho a^3 (\boldsymbol{v} \cdot \boldsymbol{n})^2, \tag{13}$$

where $\boldsymbol{v}$ is the velocity of the disk center and $\boldsymbol{n}$ is the unit vector normal to the disk plane. Disk rotation makes a small contribution to the kinetic energy, which is proportional to $\rho a^5 \omega^2$.

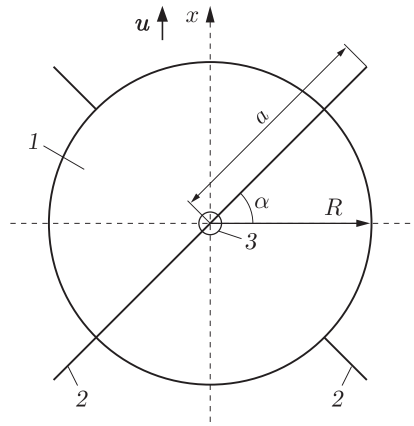

Fig. 1. Shape of a self-propelling body projected onto a plane: 1) sphere; 2) disks; 3) rod.

In the principal approximation with respect to the small parameters R/h and a/h, the kinetic energy of the fluid is determined by the formula

$$T_f = T_0 + 2T_1. \tag{14}$$

From Eqs. (12)–(14), we find the coefficient K in Eq. (3) for T_f:

$$K = -(8/3)\rho a^3 h \sin 2\alpha. \tag{15}$$

It follows from here that $K \neq 0$. We also write the expressions for two other coefficients:

$$m_f = (2/3)\pi\rho R^3 + (16/3)\rho a^3 \cos^2\alpha, \qquad I_f = (16/3)\rho a^3 h^2 \sin^2\alpha.$$

Equation (15) takes into account the three-dimensional character of the flow, which is caused by the fact that the axis of symmetry of each disk is inclined with respect to the axis of symmetry of the entire body.

If the moment of internal forces generates a moment of momentum $I_*\Omega$ inside the sphere, then the sphere with two disks moves translationally along the x axis with a velocity u determined by Eq. (9). Simultaneously, this body rotates.

Note that the body shape considered, which includes two disks and a sphere, is an analog of a ship screw. The disks play the role of the screw blades.

3. Inertial Motion of a Deformable Body. The theory considered allows the second solid to be located outside rather than inside the first body. In this case, all notations are the same. Interaction between the bodies is determined by the moment of internal forces. Formulas (2)–(11) remain valid; the above-proved theorem about the shape of an inertially moving body is also valid. The body experiences deformations owing to rotation of one solid body with respect to the other. Note, if the second body is axisymmetric, the shape of its surface remains unchanged within the framework of the model of dynamics of an ideal fluid.

4. Motion of a Body with a Rotating Screw from the State at Rest. We consider the possibility of motion of a screw-propelled ship from the state at rest in an ideal fluid. The conditions of screw operation are unusual. It is known that a rotating screw in a vortex-free flow does not generate a thrust force. We assume that the ship hull has the shape of an ellipsoid or any other shape that allows its free motion (without rotation) in the fluid along the axis of the screw. We place an inner solid interacting with the screw into the ship hull. The results of research described in Sec. 2 can be applied to solve this problem, if the moment of inertia of the screw and the shaft is denoted by I_0 and the total mass of the body is denoted by m_b. Then, the theorem proved above remains valid. Hence, we confirm the theoretical conclusion that the ship can move in an inviscid fluid owing to rotation of the screw, despite the absence of vortices in the fluid.

Conclusions. The dynamics of a heterogeneous body in a vortex-free flow of an ideal fluid in the absence of external forces is studied. The body shape providing the possibility of inertial motion of the body from the state at rest is found, namely, the theorem of the shape of a uniformly moving body is proved. An example of the body shape satisfying the theorem conditions is given, and the coefficients of the kinetic energy for this body are calculated. It is demonstrated that the body can move in an ideal fluid without vortices owing to rotation of the screw.

Uniform motion of the body from the state at rest is possible if the body surface has an axis of symmetry of the Nth order and the coefficient K at the product of translational and angular velocities of the surface $u\omega$ in Eq. (3) for the kinetic energy of the fluid is other than zero. In this case, uniform rotation of the inner solid results in inertial motion of the body in the fluid.

The following specific features of the examined motion are identified.

1. The body motion from the state at rest occurs uniformly, with a translational velocity remaining constant after a certain time.

2. The body motion continues by inertia after the action of internal forces responsible for this motion is terminated.

3. The body motion from the state at rest occurs owing to changes in only one parameter of the body: angle of rotation of the inner solid. The currently available theories predict that body displacement requires changes in at least two quantities out of the coordinates defining the shape and volume of the body and the mass distribution inside the body.

The solution obtained above implies a principal possibility of translational motion of the body by inertia owing to internal rotation. This solution involves a direct relation between inner rotation and translational motion of the body.

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